# Wyner-Ziv Estimators: Efficient Distributed Mean Estimation with Side Information 

AISTATS 2021

Prathamesh Mayekar, Indian Institute of Science
Ananda Theertha Suresh, Google
Himanshu Tyagi, Indian Institute of Science

## The Setup

## Server

Side Information $y$


Assumptions: $\left\|x_{i}-y\right\|_{2} \leq \Delta$, for all $i \in\{1, \ldots, n\}$.

## The Setup

## Server

## Side Information $y$



Assumptions: $\left\|x_{i}-y\right\|_{2} \leq \Delta$, for all $i \in\{1, \ldots, n\}$.
Server's Goal: Estimate Sample Mean $\bar{x}:=\frac{1}{n} \sum_{i=1}^{n} x_{i}$.

## The Setup

## Server



Assumptions: $\left\|x_{i}-y\right\|_{2} \leq \Delta$, for all $i \in\{1, \ldots, n\}$.
Server's Goal: Estimate Sample Mean $\bar{x}:=\frac{1}{n} \sum_{i=1}^{n} x_{i}$.
Each client can send only $r$ bits.

## The Setup

## Server



Assumptions: $\left\|x_{i}-y\right\|_{2} \leq \Delta$, for all $i \in\{1, \ldots, n\}$.
Server's Goal: Estimate Sample Mean $\bar{x}:=\frac{1}{n} \sum_{i=1}^{n} x_{i}$.
Each client can send only $r$ bits.
Two settings:

1. The known setting, where $\Delta$ is known to everyone;
2. The unknown setting, where $\Delta$ is unknown to everyone.

## Our Contributions

Prior Work: The no side information case [Suresh et al. 17]

- $\left\|x_{i}\right\|_{2} \leq 1$, for all $i \in[n]$, and no side information.
- For any $r \in[d], M S E \approx \Theta\left(\frac{d}{n r}\right)$.


## Our Contributions

Prior Work: The no side information case [Suresh et al. 17]

- $\left\|x_{i}\right\|_{2} \leq 1$, for all $i \in[n]$, and no side information.
- For any $r \in[d], M S E \approx \Theta\left(\frac{d}{n r}\right)$.


## Main Results:

1. In the known $\Delta$ setting, $M S E \approx \Theta\left(\Delta^{2} \cdot \frac{d}{n r}\right)$.

## Our Contributions

Prior Work: The no side information case [Suresh et al. 17]

- $\left\|x_{i}\right\|_{2} \leq 1$, for all $i \in[n]$, and no side information.
- For any $r \in[d], M S E \approx \Theta\left(\frac{d}{n r}\right)$.


## Main Results:

1. In the known $\Delta$ setting, $M S E \approx \Theta\left(\Delta^{2} \cdot \frac{d}{n r}\right)$.
2. In the unknown $\Delta$ setting, $M S E \approx O\left(\Delta \cdot \frac{d}{n r}\right)$.

## Our Contributions

Prior Work: The no side information case [Suresh et al. 17]

- $\left\|x_{i}\right\|_{2} \leq 1$, for all $i \in[n]$, and no side information.
- For any $r \in[d], M S E \approx \Theta\left(\frac{d}{n r}\right)$.


## Main Results:

1. In the known $\Delta$ setting, $M S E \approx \Theta\left(\Delta^{2} \cdot \frac{d}{n r}\right)$.
2. In the unknown $\Delta$ setting, $M S E \approx O\left(\Delta \cdot \frac{d}{n r}\right)$.
3. Our algorithms are nearly linear time.

## Thank You!

